An Improved Ideal Point Setting in Multiobjective Evolutionary Algorithm Based on Decomposition

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***Abstract*—In this paper, we propose an improved ideal point setting method in the framework of MOEA/D. MOEA/D decomposes a multi-objective optimisation problem into a number of scalar optimisation problems and optimise them simultaneously. The performance of MOEA/D highly relates to its decomposition method, and the proposed ideal point setting approach is used in the weighted Tchebycheff (TCH) and penalty-based boundary intersection (PBI) decomposition approach. It expands the region of search in the objective space by transforming the original ideal point into its symmetric point and changes the search direction of each subproblems in MOEA/D. In order to verify the proposed ideal point setting method, we design a set of multi-objective problems(MOPs) . The proposed method is compared with the original MOEA/D-TCH and MOEA/D-PBI on MOPs. The experimental results demonstrate that our proposed ideal point setting method performs well in terms of both diversity and convergence.**

***Index Terms*—Multi-objective Evolutionary Algorithm, Ideal Point Setting.**

I. Introduction

Engineering optimization problems usually contain more than one objectives. The objectives are often conflicting with each other, which means that improvement in one objective will lead to the degradation of at least another objective. In other words, it is impossible to make all of the objectives to be optimal simultaneously. However, a set of solutions that represent the trade-off among multiple objectives can be found. Without loss of generality, a multi-objective optimization problem can be defined as follows:

$$\begin{aligned} \text{minimize} \quad & \mathbf{F}(\mathbf{x}) = (f_1(\mathbf{x}), \dots, f_m(\mathbf{x}))^T \\ \text{subject to} \quad & \mathbf{x} \in \Omega \end{aligned} \tag{1}$$

where $\Omega = \prod_{i=1}^{n}[a_i, b_i] \subseteq \mathbb{R}^n$ is the decision space, and $\mathbf{x} = (x_1, x_2, \dots, x_n)^T \in \mathbb{R}^n$ is n-dimensional design variables, $\mathbf{F}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_m(\mathbf{x}))^T \in \mathbb{R}^m$ is m-dimensional objective vector. A solution $\mathbf{x}^1$ is said to dominate $\mathbf{x}^2$ if $f_i(\mathbf{x}^1) \leq f_i(\mathbf{x}^2)$ for each $i \in \{1, \dots, m\}$ and $f_j(\mathbf{x}^1) l f_j(\mathbf{x}^2)$ for at least one $j \in \{1, \dots, m\}$, denoted as $\mathbf{x}^1 \preceq \mathbf{x}^2$. If there is no other solution $\mathbf{x} \in \Omega$ dominating $\mathbf{x}^* \in \Omega$, $\mathbf{x}^*$ is said to be a Pareto optimal solution. The set of all the Pareto optimal solutions is called the Pareto optimal set(PS). Mapping the PS into the objective space obtains a set of objective vectors,denoted as Pareto front (PF), where $PF = \{\mathbf{F}(\mathbf{x}) \in \mathbb{R}^m | \mathbf{x} \in PS\}$. For a minimizing multi-objective optimisation problem, the minimum value of each objective in PF forms an ideal point, which can be denoted as $\mathbf{z}^* = (z_1^*, \dots, z_m^*)^T$, where $z_i^* = \min_{\mathbf{x} \in \Omega} f_i(\mathbf{x}), i \in \{1, \dots, m\}$.

Multi-objective evolutionary algorithm (MOEA) is a population-based stochastic searching algorithm, and has the capacity to solve multi-objective optimisation problems in a single run. Convergence and diversity are two important metrics in MOEA, and much effort has been made to balance them. At present, MOEAs can be classified into three categories according to their fitness assign methods, which are dominance-based, indicator-based and decomposition-based ones. In dominance-based MOEAs, the fitness of an individual is decided by non-dominated sorting and distance metrics. Representative algorithms include NSGA-II [1],SPEA-II[2], and PAES-II[3]. Indicator-based MOEAs assign the fitness values to individuals according to their contribution to the performance metrics such as IGD[4] and HV[5]. Representative algorithms include IBEA[6], R2-IBEA[7], SMS-EMOA[8] and HypE[9]. In decomposition-based MOEAs, a multi-objective optimisation problem has been decomposed into a number of single objective or simple multi-objective optimisation problems. Representative algorithms include IMMOGLS[10], UGA[11], cMOGA[12], MOEA/D[13] and MOEA/D-M2M[14]. Multi-objective evolutionary algorithm based on decomposition (MOEA/D[13]) is a very popular decomposition-based MOEA. It decomposates a multi-objective optimisation problem into a number of single objective optimisation subproblems and optimises them in a collaborative way. The diversity can be obtained in a implicit manner by setting the weight vectors in MOEA/D.A number of

MOEA/D variants have been proposed and studied (e.g., [14–18]), their main efforts focus on balancing the performance of convergence and diversity. MOEA/D-STM[16] uses a simple and effective stable matching(STM) model to coordinate the selection process in MOEA/D and it tradeoffs convergence and diversity of the evolutionary search by the STM model. MOEA/D-M2M[14] decompose a multi-objective optimisation problem into a set of simple multi-objective optimisation subproblems. Each subproblem has its own population and the diversity can be maintained by setting uniform direction vectors in the objective space. MOEA/D-DRA[17] allocates the computational resource to each subproblems dynamically according to their utility function, so that the performance of convergence can be enhanced. EAG-MOEA/D[18] uses an external archive to guide the search direction. It uses a decomposition-based strategy to evolve its working population and uses a domination-based sorting to maintain the external archive, so that the convergence and diversity can be maintained simultaneously.

The remainder of this paper is organised as follows. Section II introduces the proposed ideal point setting method and the algorithm. Section II designs a set of multi-objective optimisation problems (MOPs). Section IV gives the experimental results and Section V concludes the paper.

II. IDEAL POINT SETTING AND ALGORITHM

Ideal point is used in the decomposition methods, such as Tchebycheff (TCH) approach and Penalty-based boundary Intersection(PBI) approach. In order to address the influence of ideal point for these two decomposition methods, we will briefly introduce them and their search directions in the objective space.

1) Tchebycheff (TCH) Approach: In this method, the i_{th} decomposed single optimisation subproblems is defined as follows:

$$\begin{aligned} \text{minimize} \quad & g^{tch}(\mathbf{x}|\lambda^i, \mathbf{z}^*) = \max_{1\le j\le m}\{\lambda^i_j f_j(\mathbf{x})\} \\ \text{subject to} \quad & \mathbf{x} \in \Omega \end{aligned} \tag{2}$$

Its search direction vector a^i is defined as $(1/\lambda^i_1, \ldots, 1/\lambda^i_m)^T$

2) Penalty-Based Boundary Intersection (PBI) Approach:

$$\begin{aligned} \text{minimize} \quad & g^{pbi}(\mathbf{x}|\lambda^i, \mathbf{z}^*) = d_1 + \theta d_2 \\ & d_1 = (\mathbf{F}(\mathbf{x}) - \mathbf{z}^*)^T \lambda^i / \|\lambda^i\| \\ & d_2 = \|\mathbf{F}(\mathbf{x}) - \mathbf{z}^* - (d_1/\|\lambda^i\|)\lambda^i\| \\ \text{subject to} \quad & \mathbf{x} \in \Omega \end{aligned} \tag{3}$$

Its search direction vector a^i is defined as $(\lambda^i_1, \ldots, \lambda^i_m)^T$. Where θ is a user-defined penalty parameter, $\lambda^i = (\lambda^i_1, \ldots, \lambda^i_m)^T$ is a weighted vector, and $\sum_{j=1}^m \lambda^i_j = 1$.

In the above decomposition methods, the setting of ideal point $\mathbf{z}^*$ is critical. However, very few researchers have paid attention to it. We usually set the ideal point $\mathbf{z}^*$ to be the minimum value of each objective in the population during the evolutionary process. This method of setting ideal point $\mathbf{z}^*$ limits the search area as shown in Figure 1. In this Figure, the true PF is segment AD, if we use $\mathbf{z}^*$ as the ideal point, only the segment BC can be achieved. Because the search direction of each subproblems is limited in the area of $f_1 - \mathbf{z}^* - f_2$. However, if we move the ideal point $\mathbf{z}^*$ to $\mathbf{z}'$, the search area can be expanded to $f_1' - \mathbf{z}^* - f_2'$ in the objective space and the whole true Pareto front can be possibly achieved. The algorithm works as follows.

Algorithm:MOEA/D-IP

Input:

1) a MOP;
2) a stopping criterion;
3) N: the number of subproblems;
4) a set of of N weight vectors: $\lambda^1, \ldots, \lambda^N$;
5) T: the size of the neighborhood;
6) f: the DE parameter;
7) δ: the probability of selecting parents from the neighborhood;
8) n_r: the maximal number of solutions replaced by a child.

Output: A set of non-dominated solutions NS;

Step 1: Initialisation:

a) Decompose the MOP into N subproblems associated with $\lambda^1, \ldots, \lambda^N$.
b) Generate an initial population $P = \{\mathbf{x}^1, \ldots, \mathbf{x}^N\}$.
c) Let $NS = P$.
d) Compute the Euclidean distance between any two weight vectors and obtain T closest weight vectors to each weight vector. For each $i = 1, \ldots, N$, set $B(i) = \{i_1, \ldots, i_T\}$, where $\lambda^{i_1}, \ldots, \lambda^{i_T}$ are the T closest weight vectors to λ^i.

Step 2: Population update

For $i = 1, \ldots, N$, do

a) Generate a random number r from $[0, 1]$. If $r < \delta, S = B(i)$, else $S = \{1, \ldots, N\}$
b) Set $r_1 = i$ and randomly select two indexes from S, and generate $\mathbf{y}^i = \mathbf{x}^{r_1} + f * (\mathbf{x}^{r_2} - \mathbf{x}^{r_3})$.
c) Perform a mutation operator on $\mathbf{y}^i$, and repair $\mathbf{y}^i$.
d) Update z^*, For each $j = 1, \ldots, m$, if $z^*_j > f_j(\mathbf{y}^i)$, then set $z^*_j = f_j(\mathbf{y}^i)$.Set $z' = -z^*$.
e) **Update of Solutions**: Set $c = 0$ and then do the following:
   1) If $c = n_r$ or S is empty, go to **Step3**. Ohterwise, select an index j from S randomly.
   2) if $g(\mathbf{y}^i|\lambda^j, z') \le g(\mathbf{x}^j|\lambda^j, z')$, then set $\mathbf{x}^j = \mathbf{y}^i$ and $c = c + 1$.
   3) Remove j from S and go to 1).
f) Set $NS = P$.

Step 3: Termination If stopping criteria are satisfied, output NS. Otherwise, go to **Step 2**.

It is worth noting that the ideal point setting should consider the population's distribution. If the ideal point is too far away from the population, many subproblems will not find the optimal value in its search direction. However, if the ideal point is too close to the population, it will lose the diversity. In different evolutionary stage, the ideal point should set differently. In the early stage of the evolution, the ideal point can be set a little far away from the population and in the later evolutionary process, the ideal point should close to $\mathbf{z}^*$. In order to handle this issue, we take the symmetric point of the original ideal point $\mathbf{z}^*$ as the ideal point $\mathbf{z}^{'}$ and $\mathbf{z}^{'} = -\mathbf{z}^*$. It is worth noting that each objective of MOPs should be nonnegative, and this assumption will ensure that the position of new ideal point $\mathbf{z}^{'}$ is in the lower left of $\mathbf{z}$. Setting the new ideal point $\mathbf{z}^{'} = -\mathbf{z}^*$ will expand the search regions in the early evolutionary stage as shown in Figure 1. In the later evolutionary stage, the new ideal point $\mathbf{z}^{'}$ is very close to $\mathbf{z}^*$, and this will enhance the convergence of MOEA/D.

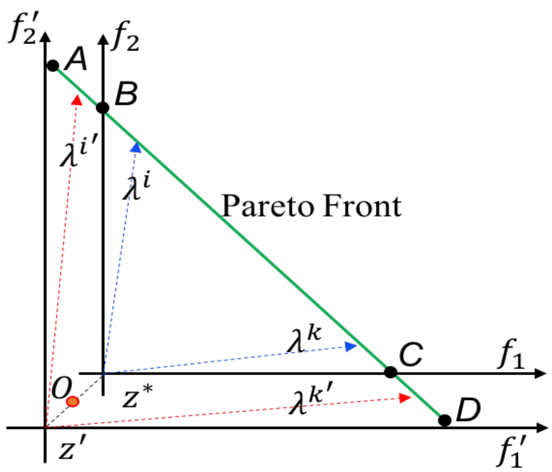

Fig. 1. Illustrations on the influence of ideal point setting.

To verify the effectiveness of the proposed ideal point setting, we design a set of multi-objective optimisation problems in the following section and conduct a series of experiments which are described in detail in the Section of Experimental Study.

III. Design of MOPs

In order to demonstrate the effectiveness of the improved ideal point setting, we will design some special multi-objective optimisation problems. These problems should have the following properties.

1) The diversity of the solutions of the multi-objective optimisation problems should not be easy to acquire.

2) The convergence of the solutions of the multi-objective optimisation problems is easy to achieve compared with their diversity.

3) The MOPs have different shapes of Pareto fronts, such as convex, concave and discrete ones.

The following MOPs are designed based on ZDT[19]. $g(x)$ functions used in our instances are different from those in their original versions. Their decision space is $[0,1]^m$, m equals 30. In order to control the difficulty level in terms of convergence, $g(x)$ should be unimodal and without deceptive optimal value. The diversity will be easy achieved if each component of decision variables in Pareto set is linear correlation. Given the above assumption, we design a set of multi-objective optimisation problems (MOPs) as shown in Table I.Every test problem has similar Pareto set, like $x_i = sin(0.5a\pi x_1)^b$, where $a \in [0,1]$ and $b \in [0,1]$.

TABLE I
Objective functions of MOP1-MOP7.

Function Name	Function Definition
MOP1 PF convex	$\begin{cases} \text{minimize} & f_1(x) = x_1 \\ \text{minimize} & f_2(x) = g(x)(1-\sqrt{f_1/g(x)}) \\ & g(x) = 1+\sum_{i=2}^{m}(x_i - sin(0.5\pi x_1))^2 \\ & m = 30, x_i \in [0,1] \end{cases}$
MOP2 PF concave	$\begin{cases} \text{minimize} & f_1(x) = x_1 \\ \text{minimize} & f_2(x) = g(x)(1-(f_1/g(x))^2) \\ & g(x) = 1+\sum_{i=2}^{m}(x_i - sin(0.5\pi x_1))^2 \\ & m = 30, x_i \in [0,1] \end{cases}$
MOP3 PF discrete	$\begin{cases} \text{minimize} & f_1(x) = x_1 \\ \text{minimize} & f_2(x) = 1+g(x)(1-\sqrt{f_1/g(x)} \\ & -(f_1/g(x))sin(10\pi f_1)) \\ & g(x) = 1+\sum_{i=2}^{m}(x_i - sin(0.5\pi x_1))^2 \\ & m = 30, x_i \in [0,1] \end{cases}$
MOP4 PF convex	$\begin{cases} \text{minimize} & f_1(x) = x_1 \\ \text{minimize} & f_2(x) = g(x)(1-\sqrt{f_1/g(x)}) \\ & t_i = x_i - \sqrt{sin(0.5\pi x_1)} \\ & g(x) = 1+\sum_{i=2}^{m} {t_i}^2 \\ & m = 30, x_i \in [0,1] \end{cases}$
MOP5 PF concave	$\begin{cases} \text{minimize} & f_1(x) = x_1 \\ \text{minimize} & f_2(x) = g(x)(1-(f_1/g(x))^2) \\ & t_i = x_i - \sqrt{sin(0.5\pi x_1)} \\ & g(x) = 1+\sum_{i=2}^{m} {t_i}^2 \\ & m = 30, x_i \in [0,1] \end{cases}$
MOP6 PF convex	$\begin{cases} \text{minimize} & f_1(x) = x_1 \\ \text{minimize} & f_2(x) = g(x)(1-\sqrt{f_1/g(x)}) \\ & t_i = x_i - (sin(0.5\pi x_1))^{0.6} \\ & g(x) = 1+\sum_{i=2}^{m} {t_i}^2 \\ & m = 30, x_i \in [0,1] \end{cases}$
MOP7 PF concave	$\begin{cases} \text{minimize} & f_1(x) = x_1 \\ \text{minimize} & f_2(x) = g(x)(1-(f_1/g(x))^2) \\ & t_i = x_i - (sin(0.5(i/m)\pi x_1))^{0.2} \\ & g(x) = 1+\sum_{i=2}^{m} {t_i}^2 \\ & m = 30, x_i \in [0,1] \end{cases}$

IV. Experimental Study

A. Experimental Settings

In order to evaluate the performance of improved ideal point setting mentioned in section II, we compare the proposed method with MOEA/D-TCH (also called MOEA/D) and MOEA/D-PBI, and then studied the experimental results on MOP1-MOP7. Thirty independent runs with the four algorithms are conducted. The detailed parameter settings are summarised as follows.

1) Setting for reproduction operators: The mutation probability $Pm = 1/n$ (n is the number of decision variables) and its distribution index is set to be 20. For the DE operator, we set $CR = 0.5$ and $f = 0.5$.

2) Population size: $N = 200$.

3) Number of runs and stopping condition: Each algorithm runs 30 times independently on each test problems. The algorithm stops until 200 000 function evaluations.

4) Neighborhood size: $T = 20$.

5) Probability use to select in the neighborhood: $\delta = 0.95$.

6) The maximal number of solutions replaced by a child: $nr = 20$.

7) The penalty parameter $\theta = 5.0$.

B. Performance Metric

In this work, the performance of a multi-objective evolutionary algorithm is evaluated in two aspects convergence and diversity. Convergence describes the closeness of the obtained Pareto front to the true Pareto front. Diversity on the other hand depicts how the solutions in the obtained Pareto are distributed. We select two metrics - inverted generation distance (IGD)[4] and relative hypervolume indicator (I_H^-)[20]. Detailed definitions of them are given as follows:

- **Inverted Generational Distance** (IGD):

Let P^* be the ideal Pareto front set, A is an approximate Pareto front set achieved by evolutionary multi-objective algorithm. IGD metric denotes the distance between P^* and A. It is defined as follows:

$$\begin{cases} IGD(P^*, A) = \frac{\sum_{y^* \in P^*} d(y^*, A)}{\|P^*\|} \\ \\ d(y^*, A) = \min_{y \in A}\{\sqrt{\sum_{i=1}^{m}(y_i^* - y_i)^2}\} \end{cases} \tag{4}$$

- **Relative Hypervolume Indicator** (I_H^-):

I_H^- simultaneously considers the distribution of the obtained Pareto front A and its vicinity to the true Pareto front. $I_H(P^*, R)$ is defined as the volume enclosed by P^* and the reference vector $R = (R_1, \ldots, R_m)$. $I_H(A, R)$ is defined as the volume enclosed by A and the reference vector R. $I_H^-(A, P^*, R)$can be defined as:

$$\begin{cases} I_H^-(A, P^*, R) = I_H(P^*, R) - I_H(A, R) \\ I_H(P^*, R) = Vol_{v \in P^*}(v) \\ I_H(A, R) = Vol_{v \in A}(v) \end{cases} \tag{5}$$

Here, $Vol_{v \in P^*}(v)$ represents the volume enclosed by solution $v \in P^*$ and the reference vector R, and $Vol_{v \in A}(v)$ represents the volume enclosed by solution $v \in A$ and the reference vector R. When computing the above metrics, 200 points are uniformly sampled from the true PF. When calculating the I_H^-, the reference point R is $(1.2, 1.2)^T$ for MOPs except MOP3, and the reference point of MOP3 is $(1.2, 2.2)^T$. The smaller values of IGD and I_H^- represent a better performance.

C. Experimental Result

In order to demonstrate the effectiveness of the proposed ideal point setting method, we compared it with MOEA/D-TCH and MOEA/D-PBI. The final populations with the best I_H^- metric in 30 independent runs in the framework of MOEA/D are shown in Figure 2.

From Figure 2, it is clear that MOEA/D-IP has obtained better Pareto fronts on MOP2, MOP5 and MOP7 than MOEA/D. For MOP1, MOP2, MOP3 and MOP4, MOEA/D and MOEA/D-IP have the similar Pareto fronts. It is worth to note that Figure 2 gives the best I_H^- metric in 30 independent runs. The mean value of IGD and I_H^- metrics are shown in Table II and Table III. The IGD metrics of MOEA/D-IP on MOP1-MOP7 are significant better than MOEA/D. In terms of I_H^- metric, MOEA/D-IP is significant better than MOEA/D on MOP1-MOP6. Although on MOP7, there is no significant difference between MOEA/D and MOEA/D-IP, the mean value of I_H^- of MOEA/D-IP is better than MOEA/D. The variance of MOEA/D-IP is much bigger than MOEA/D on MOP7, that is the reason that the I_H^- value of MOEA/D and MOEA/D-IP has no significant difference. In term of convergence speed, as shown in Figure 3, the left is IGD metric and the right is I_H^-. It can be observed that the convergence speed of MOEA/D-IP is much faster than MOEA/D on all MOPs except MOP3. However, the final IGD and I_H^- values of MOEA/D-IP is much more better than MOEA/D on MOP3.

From Figure 4, we can observe that the diversity of MOEA/D-PBI-IP is better than that of MOEA/D-PBI on MOP1, MOP2, MOP3, MOP5 and MOP7. On test instances MOP4 and MOP6, the convergence of MOEA/D-PBI-IP is better than that of MOEA/D-PBI. To further verify the effectiveness of proposed ideal point setting method, we calculate the IGD metric and I_H^- metric and make some stastic analysis, as shown in Table IV and Table V. From these two tables, we can clearly observe that MOEA/D-PBI-IP is significant better than MOEA/D-PBI in terms of both IGD metric and I_H^- metric on all of the test instances. In term of convergence speed, as shown in Figure 5, MOEA/D-PBI-IP is much better than MOEA/D-PBI on all of the test instances. From the above experimental analysis, we can conclude that the proposed ideal point setting method is very effective in both framework of MOEA/D and MOEA/D-PBI, and the effectiveness is even better in the framework of MOEA/D-PBI.

V. CONCLUSION

This paper proposes an improved ideal point setting method which transforms the original ideal point into its symmetric point in order to expand the subproblems search area in the objective space. To verify the proposed ideal point setting method, we compare it with MOEA/D using two commonly used decomposition methods - weighted Tchebycheff (TCH) and penalty-based boundary intersection (PBI). We also design a set of multi-objective optimisation problems named MOPs to verify the effectiveness of proposed ideal point setting method. Experimental results show that the proposed ideal point setting approach outperforms the original ideal setting method in terms of both convergence and diversity under the framework of MOEA/D-TCH and MOEA/D-PBI. The future work includes setting the ideal point adaptively according to the population's evolutionary information, combining the proposed ideal point setting method with other state-of-the-art algorithms to further improve their performance, and testing them in real-world applications.

TABLE II
IGD VALUES OF MOEA/D AND MOEA/D-IP.

Instance	MOEA/D		MOEA/D-IP		Wilcoxon's Rank	
–	Mean	Std.	Mean	Std.	p-value	h-value
MOP1	9.83E-02	6.00E-02	**3.09E-03**	6.28E-05	1.73E-06	**1.00E+00**
MOP2	3.18E-01	4.55E-02	**1.62E-02**	1.89E-03	1.73E-06	**1.00E+00**
MOP3	1.67E-01	8.40E-02	**8.97E-02**	1.19E-01	2.43E-02	**1.00E+00**
MOP4	4.47E-03	1.91E-03	**2.87E-03**	9.38E-05	1.73E-06	**1.00E+00**
MOP5	5.03E-01	3.84E-02	**2.71E-02**	8.61E-02	1.73E-06	**1.00E+00**
MOP6	2.46E-02	4.52E-02	**2.85E-03**	6.70E-05	1.73E-06	**1.00E+00**
MOP7	5.97E-01	1.87E-02	**3.66E-01**	2.80E-01	4.45E-05	**1.00E+00**

TABLE III
I_H^- VALUES OF MOEA/D AND MOEA/D-IP.

Instance	MOEA/D		MOEA/D-IP		Wilcoxon's Rank	
–	Mean	Std.	Mean	Std.	p-value	h-value
MOP1	1.02E-01	5.62E-02	**2.85E-03**	1.13E-04	1.73E-06	**1.00E+00**
MOP2	4.13E-01	3.65E-02	**8.27E-03**	8.88E-04	1.73E-06	**1.00E+00**
MOP3	4.10E-01	1.77E-01	**1.95E-01**	2.73E-01	1.48E-03	**1.00E+00**
MOP4	5.55E-03	2.88E-03	**3.44E-03**	2.97E-04	9.32E-06	**1.00E+00**
MOP5	5.14E-01	1.45E-02	**2.41E-02**	9.16E-02	1.73E-06	**1.00E+00**
MOP6	2.78E-02	4.86E-02	**2.94E-03**	1.34E-04	1.73E-06	**1.00E+00**
MOP7	5.30E-01	8.81E-04	**3.38E-01**	2.59E-01	1.02E-01	0.00E+00

TABLE IV
IGD VALUES OF MOEA/D-PBI AND MOEA/D-PBI-IP.

Instance	MOEA/D-PBI		MOEA/D-PBI-IP		Wilcoxon's Rank	
–	Mean	Std.	Mean	Std.	p-value	h-value
MOP1	3.06E-01	3.90E-02	**5.65E-03**	1.74E-04	1.73E-06	**1.00E+00**
MOP2	5.69E-01	5.84E-02	**9.38E-03**	4.27E-04	1.73E-06	**1.00E+00**
MOP3	2.60E-01	3.12E-04	**6.87E-02**	1.21E-01	1.36E-05	**1.00E+00**
MOP4	3.54E-01	3.77E-02	**5.28E-03**	2.31E-04	1.73E-06	**1.00E+00**
MOP5	6.10E-01	2.87E-04	**8.71E-03**	4.05E-04	1.73E-06	**1.00E+00**
MOP6	3.49E-01	4.43E-02	**5.36E-03**	2.12E-04	1.73E-06	**1.00E+00**
MOP7	6.10E-01	5.36E-05	**6.85E-03**	3.79E-04	1.73E-06	**1.00E+00**

TABLE V
I_H^- VALUES OF MOEA/D-PBI AND MOEA/D-PBI-IP.

Instance	MOEA/D-PBI		MOEA/D-PBI-IP		Wilcoxon's Rank	
–	Mean	Std.	Mean	Std.	p-value	h-value
MOP1	2.75E-01	3.30E-02	**8.05E-03**	3.05E-04	1.73E-06	**1.00E+00**
MOP2	5.24E-01	1.69E-02	**1.21E-02**	5.88E-04	1.73E-06	**1.00E+00**
MOP3	5.95E-01	5.09E-04	**1.39E-01**	2.58E-01	1.36E-05	**1.00E+00**
MOP4	3.21E-01	3.33E-02	**9.08E-03**	7.03E-04	1.73E-06	**1.00E+00**
MOP5	5.31E-01	1.33E-08	**1.14E-02**	1.12E-03	1.73E-06	**1.00E+00**
MOP6	3.15E-01	3.83E-02	**8.82E-03**	5.77E-04	1.73E-06	**1.00E+00**
MOP7	5.31E-01	1.82E-05	**1.03E-02**	6.07E-04	1.73E-06	**1.00E+00**

ACKNOWLEDGMENT

This research work was supported by Guangdong Key Laboratory of Digital Signal and Image Processing, the National Natural Science Foundation of China under Grant (61175073, 61300159, 61332002, 51375287), Jiangsu Natural Science Foundation (BK20130808) and Science and Technology Planning Project of Guangdong Province, China (2013B011304002).

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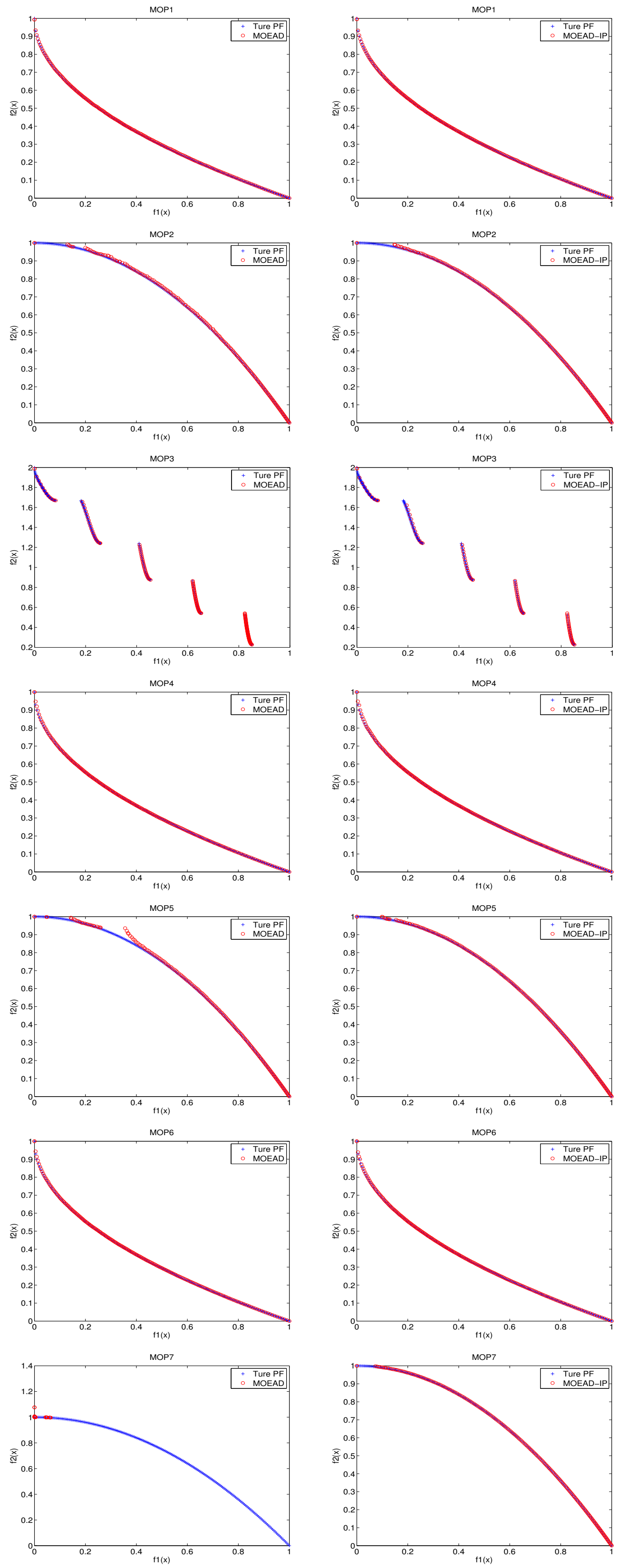

Fig. 2. The final populations with the best I_H^- metric in 30 independent runs using MOEA/D and MOEA/D-IP

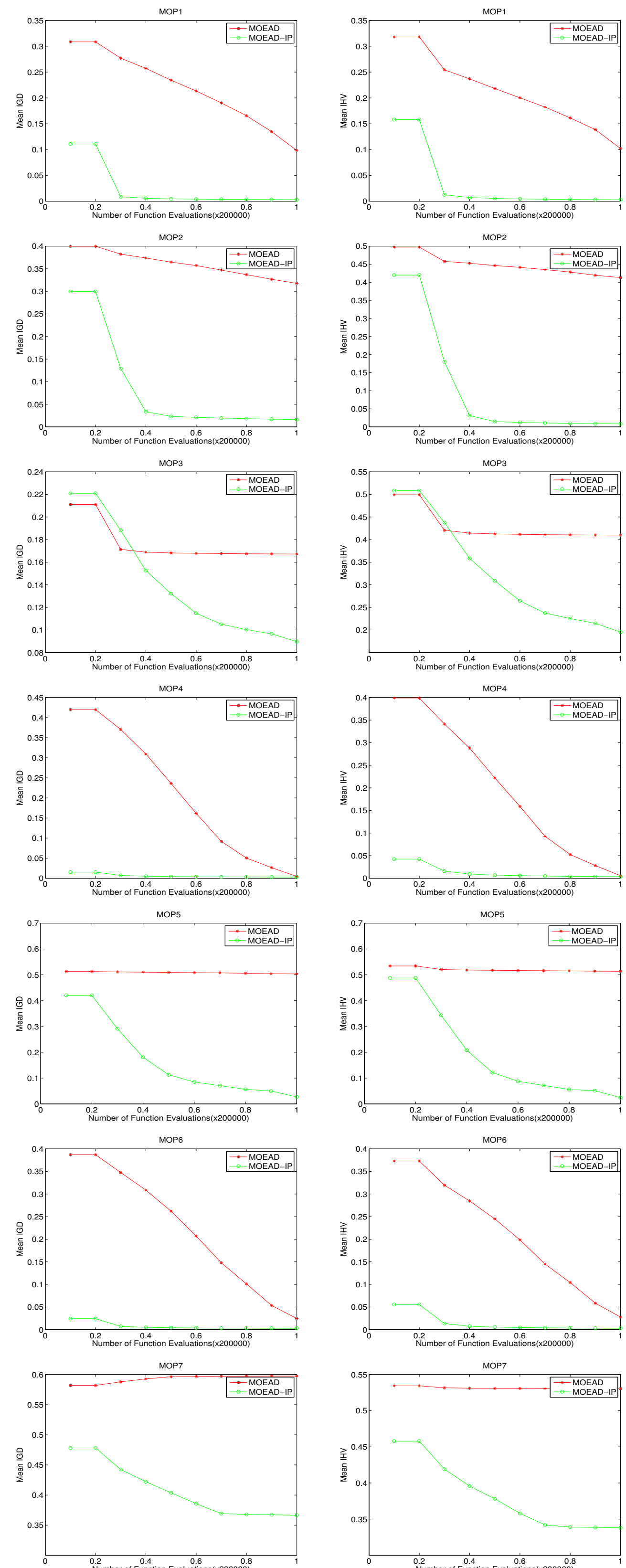

Fig. 3. The curve of mean IGD and I_H^- in 30 independent runs using MOEA/D and MOEA/D-IP

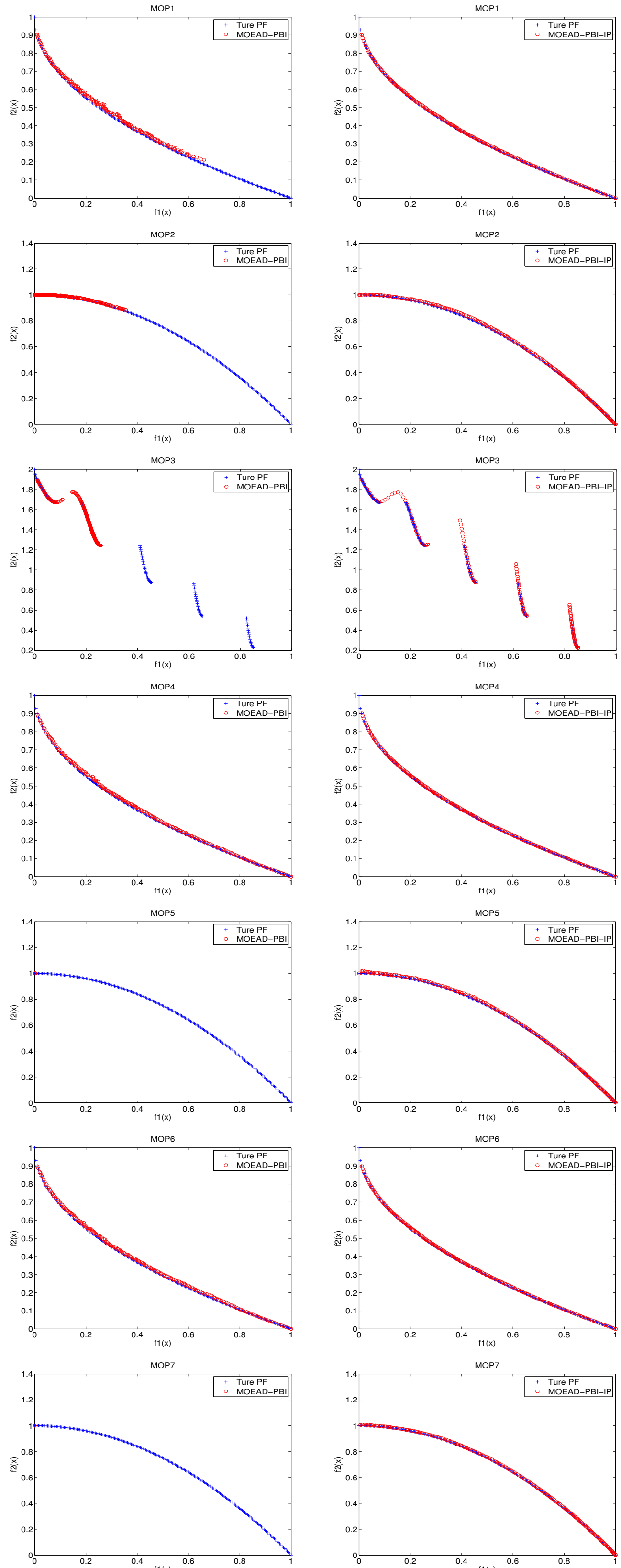

Fig. 4. The final populations with the best I_H^- metric in 30 independent runs using MOEA/D-PBI and MOEA/D-PBI-IP

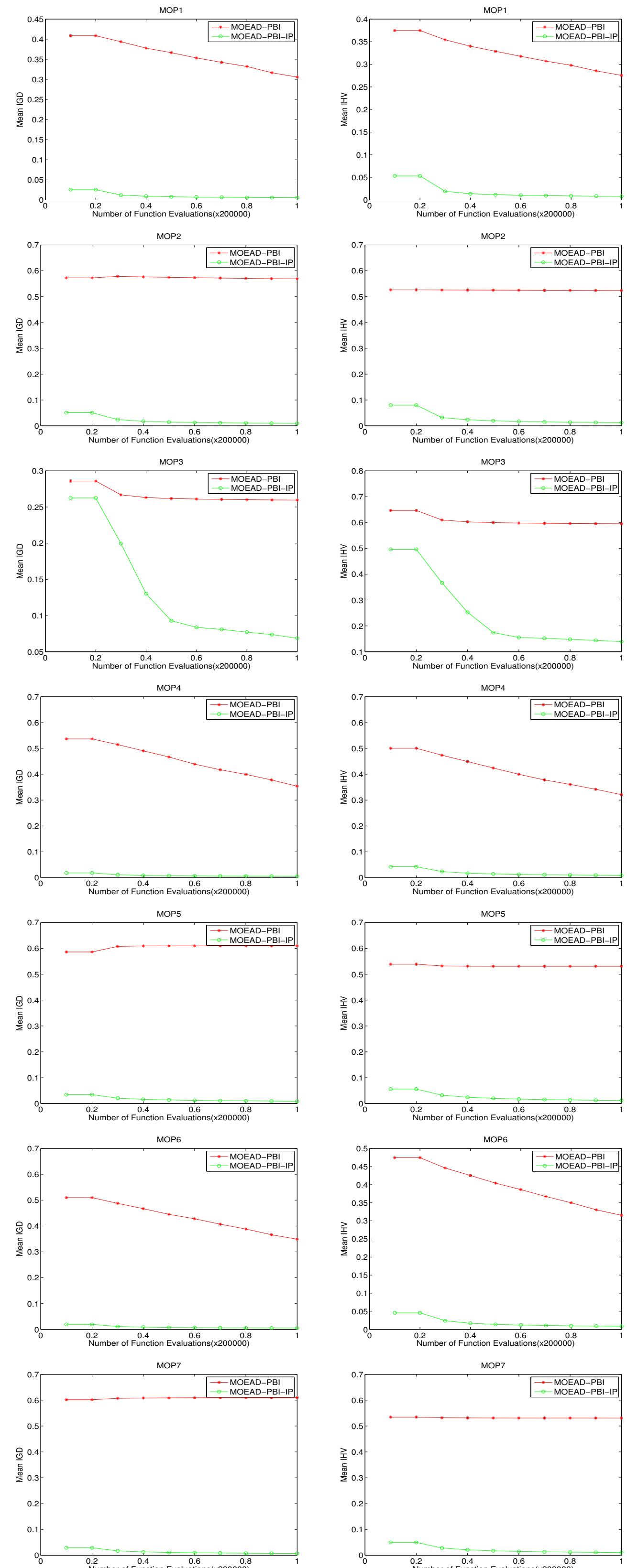

Fig. 5. The curve of mean IGD and I_H^- in 30 independent runs using MOEA/D-PBI and MOEA/D-PBI-IP

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